A group G is a set of elements $(a, b, c, \ldots)$ with a law for combining any two elements a, b so as to form an ordered product ab, such that the following 4 conditions hold:

i) For every $a, b \in G$, the product $ab \in G$

ii) The law of combination is associative, i.e.

$$(ab)c = a(bc)$$

iii) G contains a unique identity element e, such that for all $a \in G$,

$$ae = ea = a$$

iv) For all $a \in G$, there is a unique inverse element, a^{-1}, such that

$$aa^{-1} = a^{-1}a = e$$

Exercise 1.37

Do the canonical transformations on a system comprise a group? Explain your answer.

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i) is satisfied, $C_3 = C_1 \otimes C_2$, we had this in theory

ii) we need to check if $(C_1 \otimes C_2) \otimes C_3 = C_1 \otimes (C_2 \otimes C_3)$

defining $(q, p) \xrightarrow{C_1} (q', p')$,

$(q', p') \xrightarrow{C_2} (q'', p'')$

$(q'', p'') \xrightarrow{C_3} (q''', p''')$

we have

$$C_1 \otimes C_2 \Rightarrow (q, p) \xrightarrow{C_1 \otimes C_2} (q'', p'')$$

$$(q, p) \xrightarrow{(C_1 \otimes C_2) \otimes C_3} (q''', p''')$$

on the other hand

$$(q', p') \xrightarrow{C_2 \otimes C_3} (q''', p''')$$

$$(q, p) \xrightarrow{C_1 \otimes (C_2 \otimes C_3)} (q''', p''')$$

So ii) is satisfied

iii) there is an identity transformation

$$(q, p) \xrightarrow{I} (q, p)$$

iv) we have inverse transformations

$$(q', p') \xrightarrow{C_1^{-1}} (q, p)$$

So, canonical transformations do comprise a group.

Typo
Eq 8.3