A group G is a set of elements $(a, b, c, \dots)$ with a law for combining any two elements a, b so as to form an ordered product ab, such that the following 4 conditions hold:

i) For every $a, b \in G$, the product $ab \in G$

ii) The law of combination is associative, i.e.

$$(ab)c = a(bc)$$

iii) G contains a unique identity element e, such that for all $a \in G$,

$$ae = ea = a$$

iv) For all $a \in G$, there is a unique inverse element, a^{-1}, such that

$$aa^{-1} = a^{-1}a = e$$

— — — —

Exercise 1.37

Do the canonical transformations on a system comprise a group? Explain your answer.

— — —

i) is satisfied, $C_3 = C_1 \otimes C_2$, we had this in theory

ii) we need to check if $(C_1 \otimes C_2) \otimes C_3 = C_1 \otimes (C_2 \otimes C_3)$

defining
$$(q, p) \xrightarrow{C_1} (q', p'),$$
$$(q', p') \xrightarrow{C_2} (q'', p'')$$
$$(q'', p'') \xrightarrow{C_3} (q''', p''')$$

we have

$$C_1 \otimes C_2 \Rightarrow (q, p) \xrightarrow{C_1 \otimes C_2} (q'', p'')$$
$$(q, p) \xrightarrow{(C_1 \otimes C_2) \otimes C_3} (q''', p''')$$

on the other hand

$$(q', p') \xrightarrow{C_2 \otimes C_3} (q''', p''')$$
$$(q, p) \xrightarrow{C_1 \otimes (C_2 \otimes C_3)} (q''', p''')$$

So ii) is satisfied