Exercise 1.31

a) Construct a generating function $G_2(q^N, p^N)$ that gives transformation:

$$q_1' = \alpha q_1 \qquad q_n' = q_n$$
$$p_1' = \frac{1}{\alpha} p_1 \qquad p_n' = p_n \qquad , \quad 2 \le n \le N$$

b) Show explicitly that the Jacobian of this transformation is unity.

$$p_i = \frac{\partial G_2}{\partial q_i} , \qquad q_i' = \frac{\partial G_2}{\partial p_i'}$$

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$$G_2 = \alpha q_1 p_1' + \sum_n q_n p_n'$$

So this part is simple.
Now, to calculate the Jacobian.

$$J\left(\frac{q', p'}{q, p}\right) = \begin{vmatrix} \frac{\partial q_1'}{\partial q_1} & \frac{\partial q_2'}{\partial q_1} & \cdots & \frac{\partial p_1'}{\partial q_1} & \cdots & \frac{\partial p_N'}{\partial q_1} \\ \vdots & \vdots & & \vdots & & \vdots \\ \frac{\partial q_1'}{\partial p_N} & \frac{\partial q_2'}{\partial p_N} & \cdots & \frac{\partial p_1'}{\partial p_N} & \cdots & \frac{\partial p_N'}{\partial p_N} \end{vmatrix}$$

So we will have

$$\begin{vmatrix} \alpha & & & & & 0 \\ & 1 & & & & \\ & & 1 & & & \\ & & & \ddots & & \\ & & & & \frac{1}{\alpha} & \\ 0 & & & & & 1 \; 1 \; \ddots \end{vmatrix} = 1$$

I will hold that this is proven.